1997-98 KIRIS ASSESSMENT
Open-Response Item Scoring Worksheet

Grade 11—Mathematics Question 12

The **academic expectation** addressed by "Interpreting Plant Growth" (Question 12) is

2.8 Students understand various mathematical procedures and use them appropriately.

The **core content** assessed by this item includes

Algebraic Ideas

Concepts

- Students should understand systems of equations and their representations.

Skills

- Students should be able to solve and graph a variety of equations and inequalities.

Interpreting Plant Growth

The equations below show the growth of two plants, with **x** representing the growing time in weeks and **y** representing the height of the plant in inches.

Plant 1: y = 1x + 3
Plant 2: y = 3x + 3

a. Using the grid in your Student Response Booklet, graph each of the above equations. Label the y-axis "height" and the x-axis "time".

b. Find the point of intersection of the graphs of the two equations and explain what the graphs tell you about the growth of each of the two plants. Include in your explanation a description of the significance of both the y-coordinate of the intersection and the slopes of the graphs of the equations for Plant 1 and Plant 2.

c. Algebraically solve the system of equations.

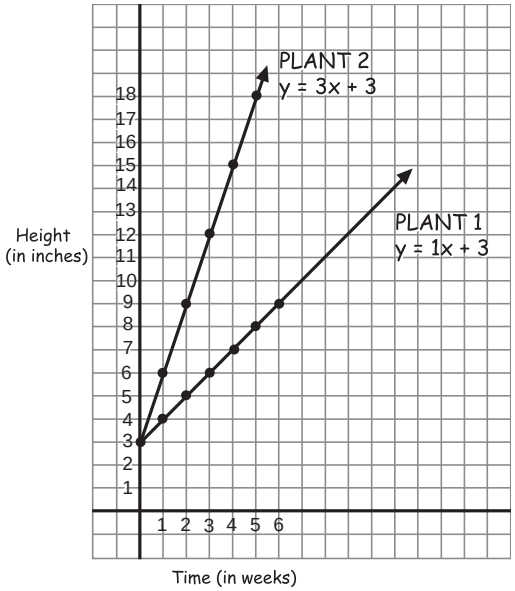
SCORING GUIDE
Grade 11 Mathematics

Score	Description
4	Student scores 6 points.
3	Student scores 4 or 5 points.
2	Student scores 2 or 3 points. **OR** Student shows an ability to perform some correct procedure(s) in two or three parts.
1	Student scores 1 point. **OR** Student shows some understanding of how to graph an equation or how to interpret the graph of an equation or how to interpret linear equations.
0	Response is totally incorrect or irrelevant.
Blank	No response.

Correct answer:

part a 2 points for correct graph for both equations y = 1x + 3; y = 3x + 3
(with correctly labeled axes: y axis = height, x axis = time)

OR



1 point for correct graph for both equations without axes labeled

1 point for correct graph for one equation (with or without axes labeled)

part b	1 point	names correct coordinates of the point of intersection (0,3)
	1 point	gives appropriate description of y-coordinate of intersection–both plants were three inches tall or both plants were the same height
	1 point	gives appropriate description of slope–describes the rate of growth. Plant 1 grows one inch per week; Plant 2 grows three inches per week. Plant 2 grows three times as fast as Plant 1.
part c	1 point	for correct algebraic solution

Plant 1	Plant 2
$y = x + 3$	$y = 3x + 3$
$(3x + 3) = x + 3$	$y = 3(0) + 3$
$2x + 3 = 3$	$y = 3$
$2x = 0$	
$x = 0$	

Correct answers:

Sample points students might plot when graphing:

Plant 1: $y = 1x + 3$

x	y
0	3
1	4
2	5
3	6
4	7

Plant 2: $y = 3x + 3$

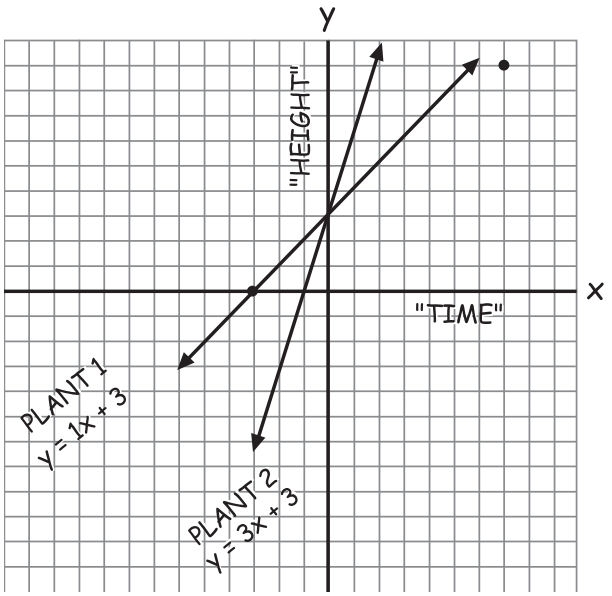
x	y
0	3
1	6
2	9
3	12
4	15

Most students usually use a scale of 1 for both the x-axis and y-axis. Some may choose a different scale.

ANNOTATED STUDENT RESPONSE
Grade 11 Mathematics

Sample 4-Point Response of Student Work

Student Response



b. The point of intersection for the equations y = 1x + 3 and y = 3x + 3 is (0,3).

The y-coordinate is significant because it shows me at the beginning of the experiment, both plants were 3 inches tall. I know this because y is the height and x is the time. Here, the x value is 0, showing that the experiment had not begun yet.

Also, quadrants II and III on the coordinate plane in this case are not valid information. Time can not be represented by a negative value, nor can height.

The slopes of the two graphs show me that Plant 2 grew taller in a faster amount of time than Plant 1. I can see this by the graph or simply by looking at the equations.

P.1 - y = 1x + 3

P.2 - y = 3x + 3

← Student correctly graphs each equation, correctly labels the vertical and horizontal axes, correctly labels the line for each equation, and appropriately scales the axes.

← Student correctly identifies (0,3) as the point of intersection for the graphs of the two equations.

← Student correctly interprets the significance of the y-coordinate of the intersection (i.e., "both plants were 3 inches tall").

← Student correctly points out that the graphed lines extend into quadrants that do not represent valid information (i.e., the information does not relate to the context of this problem).

← Student correctly and clearly explains the significance of the slopes of the two equations (i.e, Plant 2 grew at a faster rate than Plant 1).

(Sample 4-Point Response of Student Work continued on next page)

Sample 4-Point Response of Student Work (Continued)

Student Response Continued

The value before x (commonly known as b) is larger in the equation for Plant 2.

c.

$1x + 3 = 3x + 3$

$-1(1x + 3) = 3x + 3$

$-1x - 3 = 3x + 3$

$2x = 0$

$\frac{2x}{2} = \frac{0}{2}$

$x = 0$

$y = 1(0) + 3$

$y = 0 + 3$

$y = 3$

$(0,3)$ ← pt. of intersection

← Student uses algebra correctly to solve the system of equations (i.e., x = 0, y = 3). Student first reduces the system to one equation and one unknown (x), then solves for x, and then for y.

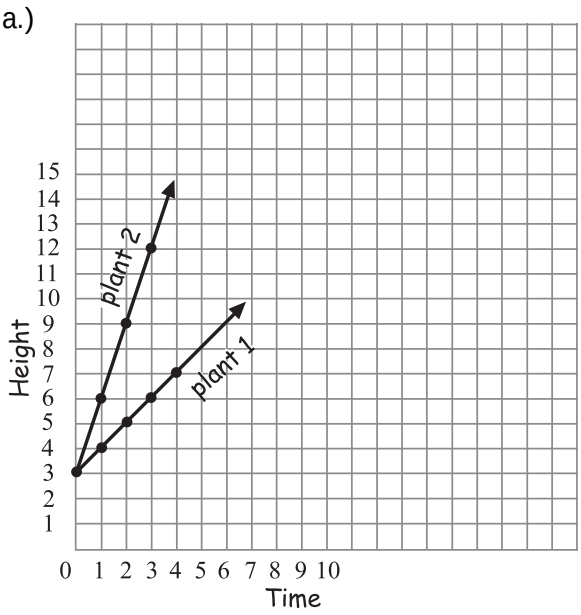
Overall, student receives a score of 4 for correctly answering part a, part b, and part c of the item. Student accurately graphs a system of two equations that represent the growth of two plants, clearly explains what the graphs tell about the growth of each of the two plants, and correctly uses algebra to solve the system of equations.

ANNOTATED STUDENT RESPONSE
Grade 11 Mathematics

Sample 3-Point Response of Student Work

Student Response

a.)



By just looking at the equations and looking at the graph, I can tell you that the point of intersection is at (0,3). In the equation they both have the same Y-intercept. The graph tells me, using the point of intersection, that the 2 plants grew evenly until they reached 3 in. Then plant 2 began growing must faster. This is depicted on the graph plus the slope of plant 2 is 3 to 1 greater than that of Plant 1. I know that because in this type of equation the number in front of the X is the slope.

In solving this system of equations, I found out algebraically that the point of intersection is (0,3).

c. $-3(y = 1x + 3$

$Y = 3x + 3$

$-3y = -3x - 9$

$y = 3x + 3$

$\frac{-2y}{-2} = \frac{-6}{-2}$

$y = 3$

$y = 3x + 3$

$3 = 3x + 3$

$-3 \quad -3$

$\frac{0}{3} = \frac{3x}{3}$

$0 = x$

← The student correctly graphs each equation, correctly labels the vertical and horizontal axes, correctly labels the line for each equation, and appropriately scales the axes.

← Student accurately identifies (0,3) as the point of intersection for the graphs of the two equations but partially misinterprets the significance of the y-coordinate of the intersection (i.e., student correctly implies that each plant was 3 inches at the y-coordinate, but mistakenly states that the plants grew at the same rate until they reached 3 inches).

← Student correctly describes Plant 2's rate of growth as three times that of Plant 1 based on the slopes of the graphs.

← Student uses algebra correctly to solve the system of equations (i.e., x = 0, y = 3).

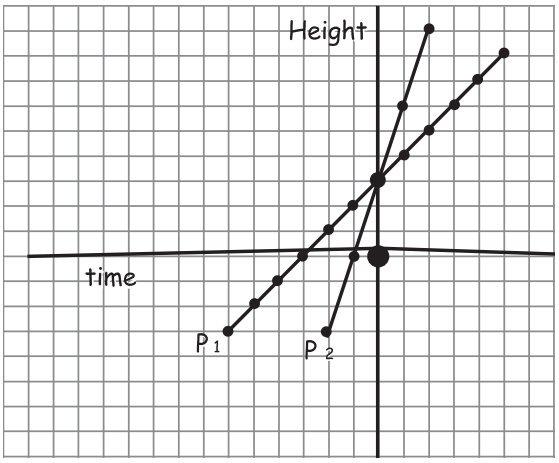
Overall, student receives a score of 3 for correctly answering part a, part c, and most of part b.

ANNOTATED STUDENT RESPONSE
Grade 11 Mathematics

Sample 2-Point Response of Student Work

Student Response

a.)



b. Plant one grows slower than plant two. They higher the slope the steaper the plant grows.

C. Y = 1x + 3 y - 3 = x

y = 3x + 3

y = (y-3) - 3

y

Student correctly graphs each equation (although, technically, student should not have extended the lines into the negative quadrants or should have explained that those quadrants do not relate to the problem). Student appropriately scales the axes, and correctly labels the vertical and horizontal axes and the line for each equation.

Student correctly explains that Plant 1 grows slower than Plant 2, but incorrectly interprets the meaning of the slope. Student does not identify the point of intersection of the graphs of the two equations or the significance of the y-coordinate of the intersection.

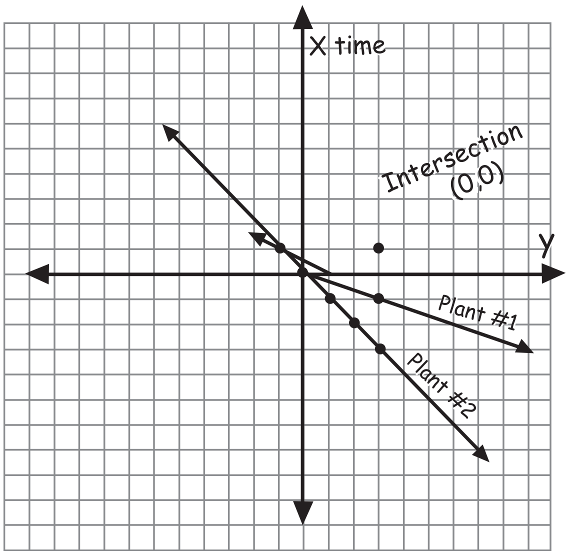
Student attempts to use algebra to solve the system of equations but the work is incomplete and shows no solutions.

Overall, student receives a score of 2 for correctly answering part a and some of part

Sample 1-Point Response of Student Work

Student Response

a.)



β). Τηε σεχονδ παρτ ισ ανδ λινεσ οφ ιντερσεχτιον τελλ μετηατ βοτη πλαντσ γροω ατ α δοωνωορδ σλοπε. Ον τηε γραπη τηε Ξ αξισ ισ γοινγ υπ ανδ δοων ρεπρεσεντινγ τιμε. (↑↓)

The "y" axis is representing the plants height (⇄).

The slopes of each line are clearly shown by a label in the down end of each slope. The intersection of both plants occured at the points (0,0). This represents the time & height at which both plants were the same.

C. plant #1

$y = 1x + 3$

$\underline{-3 \qquad -3}$

$\frac{3y}{3} = \frac{1x}{3}$

$y = \frac{1x}{3}$

plant #2

$y = 3x + 3$

$\underline{-3 \qquad -3}$

$\frac{-3y}{-3} = \frac{3x}{-3}$

$y = \frac{-1}{1}$

The student incorrectly graphs each equation. Student incorrectly uses the horizontal axis to plot the y-values and the vertical axis to plot the x-values (time).

Student clearly labels each of the line graphs and scales the axes appropriately.

Student provides an inaccurate explanation of the significance of the slopes of the equations' graphs.

Student incorrectly identifies (0,0) as the point of intersection of the two equations; however, student's interpretation of the meaning of the point of intersection is correct (i.e., the point of intersection is the point at which both plants are the same height).

Student attempts to solve the system of equations algebraically; however, the algebra and the solution are incorrect.

Student receives a score of 1 for correctly explaining one part of part b (i.e., the significance of the point of intersection).

INSTRUCTIONAL STRATEGIES
Grade 11 Mathematics

The open-response item "Interpreting Plant Growth" is designed to assess students' (1) understanding of systems of equations, (2) ability to solve and graph a system of equations, and (3) ability to interpret and describe the graph of a system of equations. The instructional strategies below present ideas for helping students practice and master these concepts and skills.

Review the concept of line graphs (i.e., the graphing of equations of lines). Demonstrate a variety of methods used for graphing lines (e.g., point-slope, two or more points).

Review the concept of slope (rate of change). Demonstrate how to determine the slope from equations, graphs, and points.

Review the meaning of "system of equations." Show a variety of problems that can be solved using a system of equations. Demonstrate different methods for solving systems of equations (e.g., substitution, elimination, graphing).

Teach students a variety of strategies for organizing information (e.g., tables, charts, graphs, highlighting, underlining) that they can use to solve problems, make comparisons, or show relationships. Explain to students that such strategies can help them to write equations, check their reasoning and the reasonableness of their answers, document their thinking, and explain their work to others.

Provide opportunities for students to work individually, in pairs, and/or in small groups to complete (with teacher support and guidance) any or all of the following activities:

- Construct graphs of equations of lines.
- Determine the slope and intercepts of lines both mathematically and graphically.
- Examine a variety of two-variable data sets and discuss appropriate scales and labels for graphical representation.
- Examine a variety of graphs and interpret the meaning of the graphs both within and outside the domain of the data sets. Discuss and write about the meaning of the graphs. (This activity can help students develop and/or refine their ability to effectively communicate their mathematical thinking both verbally and in writing.)
- Use a variety of methods (e.g., graphing, substitution, elimination) to solve systems of equations.
- Use equations and systems to solve real-world problems.